

Simulation Based on Michel Fodje's epr-simple simulation translated from Python to Mathematica by John Reed 13 Nov 2013 and Quaternions Modified by Fred Diether for Completely Local-Realistic Sep 2021 Some parts by Bill Nelson. Includes Joy's S^3 Quaternion Model.

Load Quaternion Package, Set Run Time Parameters, Initialize Arrays and Tables

```
In[147]:= << Quaternions` ;
 $\beta_0$  = Quaternion[1, 0, 0, 0];
 $\beta_1$  = Quaternion[0, 1, 0, 0];
 $\beta_2$  = Quaternion[0, 0, 1, 0];
 $\beta_3$  = Quaternion[0, 0, 0, 1];
Qcoordinates = { $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ };
m = 2000000;
trialDeg = 721;
Ls1 = ConstantArray[0, m];
Ls2 = ConstantArray[0, m];
 $\lambda_1$  = ConstantArray[0, m];
 $\lambda_2$  = ConstantArray[0, m];
qa = ConstantArray[0, m];
aq1 = ConstantArray[0, m];
qa1 = ConstantArray[0, m];
qb = ConstantArray[0, m];
bq1 = ConstantArray[0, m];
qb1 = ConstantArray[0, m];
aa1 = ConstantArray[0, m];
bb1 = ConstantArray[0, m];
outA1 = Table[{0, 0, 0, 0}, m];
outA2 = Table[{0, 0, 0, 0}, m];
outB1 = Table[{0, 0, 0, 0}, m];
outB2 = Table[{0, 0, 0, 0}, m];
plotq = Table[{0, 0}, m];
a1 = ConstantArray[0, m];
b1 = ConstantArray[0, m];
A = ConstantArray[0, m];
B = ConstantArray[0, m];
nPP = ConstantArray[0, trialDeg];
nNN = ConstantArray[0, trialDeg];
nPN = ConstantArray[0, trialDeg];
nNP = ConstantArray[0, trialDeg];
nAP = ConstantArray[0, trialDeg];
nBP = ConstantArray[0, trialDeg];
nAN = ConstantArray[0, trialDeg];
nBN = ConstantArray[0, trialDeg];
 $\phi$  = 3;  $\beta$  = 0.32;  $\xi$  = 0.933; (*Adjustable parameters for fine tuning*)
```

Generating Particle Data with Three Independent Do-Loops

```

In[185]:= Do[ $\theta = \text{RandomReal}[\{-179, 180\}];$  (*Singlet vector angle*) (*Hidden Variable*)
   $\lambda 1[[i]] = \beta \left( \text{Cos}\left[\frac{\theta}{3}\right]^2\right);$ 
   $\lambda 2[[i]] = \xi \left( \text{Cos}\left[\frac{\theta}{3}\right]^2\right);$ 
   $\theta\theta = \text{N}[\text{Flatten}[\{\text{FromPolarCoordinates}[\{1, \theta * \pi/180\}], \theta\}]];
  \text{Ls1}[[i]] = \theta\theta.\text{Qcoordinates};
  \text{Ls2}[[i]] = -\theta\theta.\text{Qcoordinates}, \{i, m\}]

In[186]:= Do[ $a = \text{RandomInteger}[\{-179, 180\}];$  (*Detector vector angle 1 degree increments*)
   $aa1[[i]] = a;$ 
   $aa = \text{N}[\text{Flatten}[\{\text{FromPolarCoordinates}[\{1, a * \pi/180\}], \theta\}]];
  \text{Da} = aa.\text{Qcoordinates};$  (*Convert to quaternion coordinates*)
   $qa = \text{Da} ** \text{Ls1}[[i]];$ 
   $qa1[[i]] = qa;$ 
   $aq = -(-\text{Da} ** \text{Ls1}[[i]]);$ 
   $aq1[[i]] = aq;$ 
   $\text{If}[\text{Abs}[\text{Re}[qa]] > \lambda 1[[i]], \text{Aa1} = \text{Sign}[\text{Re}[qa]], \text{Aa1} = \text{Re}[aq]];
  \text{outA1}[[i]] = \{a, \text{Aa1}, i, qa\}, \{i, m\}]
  Do[ $\text{If}[\text{Abs}[\text{Re}[qa1[[i]]]] < \lambda 2[[i]], \text{Aa2} = -\text{Sign}[qa1[[i]][[4]]], \text{Aa2} = \text{Re}[aq1[[i]]]];
  \text{outA2}[[i]] = \{aa1[[i]], \text{Aa2}, i + m, qa1[[i]]\}, \{i, m\}]
  \text{outA} = \text{Catenate}[\{\text{outA1}, \text{outA2}\}];

In[189]:= Do[ $b = \text{RandomInteger}[\{-179, 180\}];$  (*Detector vector angle 1 degree increments*)
   $bb1[[i]] = b;$ 
   $bb = \text{N}[\text{Flatten}[\{\text{FromPolarCoordinates}[\{1, b * \pi/180\}], \theta\}]];
  \text{Db} = bb.\text{Qcoordinates};$  (*Convert to quaternion coordinates*)
   $qb = \text{Ls2}[[i]] ** \text{Db};$ 
   $qb1[[i]] = qb;$ 
   $bq = -(-\text{Ls2}[[i]] ** \text{Db});$ 
   $bq1[[i]] = bq;$ 
   $\text{If}[\text{Abs}[\text{Re}[qb]] > \lambda 1[[i]], \text{Bb1} = \text{Sign}[\text{Re}[qb]], \text{Bb1} = \text{Re}[bq]];
  \text{outB1}[[i]] = \{b, \text{Bb1}, i, qb\}, \{i, m\}]
  Do[ $\text{If}[\text{Abs}[\text{Re}[qb1[[i]]]] < \lambda 2[[i]], \text{Bb2} = \text{Sign}[qb1[[i]][[4]]], \text{Bb2} = \text{Re}[bq1[[i]]]];
  \text{outB2}[[i]] = \{bb1[[i]], \text{Bb2}, i + m, qb1[[i]]\}, \{i, m\}]
  \text{outB} = \text{Catenate}[\{\text{outB1}, \text{outB2}\}];$$$$$ 
```

Statistical Analysis of the Particle Data Received from Alice and Bob

```

In[192]:=  $m2 = 2 m;$ 
 $\text{theta} = \text{ConstantArray}[\theta, m2];$ 
 $a1 = \text{outA}[\text{All}, 1];$   $A = \text{outA}[\text{All}, 2];$   $b1 = \text{outB}[\text{All}, 1];$   $B = \text{outB}[\text{All}, 2];$ 
Do[ $\text{th} = a1[[i]] - b1[[i]] + 361;$ 
  (*All angles are shifted by 361 degrees since  $\theta 1$  is an index*)
   $\text{theta}[[i]] = \text{th};$ 
   $\text{aliceD} = A[[i]]; \text{bobD} = B[[i]];
  \text{If}[\text{aliceD} == 1, \text{nAP}[\text{th}] ++];
  \text{If}[\text{bobD} == 1, \text{nBP}[\text{th}] ++];
  \text{If}[\text{aliceD} == -1, \text{nAN}[\text{th}] ++];
  \text{If}[\text{bobD} == -1, \text{nBN}[\text{th}] ++];
  \text{If}[\text{aliceD} == 1 \&\& \text{bobD} == 1, \text{nPP}[\text{th}] ++];
  \text{If}[\text{aliceD} == 1 \&\& \text{bobD} == -1, \text{nPN}[\text{th}] ++];
  \text{If}[\text{aliceD} == -1 \&\& \text{bobD} == 1, \text{nNP}[\text{th}] ++];
  \text{If}[\text{aliceD} == -1 \&\& \text{bobD} == -1, \text{nNN}[\text{th}] ++], \{i, m2\}]$ 
```

Calculating Mean Values of A, B, and AB, and Plotting the Results

```

In[196]:= mean = ConstantArray[0, trialDeg];
sum1 = ConstantArray[0, trialDeg];
sum2 = ConstantArray[0, trialDeg];
Do[sum1[[i]] = (nPP[[i]] + nNN[[i]] - nPN[[i]] - nNP[[i]]);
  sum2[[i]] = nPP[[i]] + nPN[[i]] + nNP[[i]] + nNN[[i]] + 0.0000001;
  mean[[i]] = sum1[[i]]/sum2[[i]], {i, trialDeg}]

In[200]:= simulation = ListPlot[mean, PlotMarkers -> {Automatic, Tiny}];
negcos = Plot[-Cos[x Degree], {x, 0, 720}, PlotStyle -> {Magenta}, AspectRatio -> 7/16,
  Ticks -> {{0, -360}, {90, -270}, {180, -180}, {270, -90}, {360, 0}, {450, 90},
    {540, 180}, {630, 270}, {720, 360}}, Automatic, GridLines -> Automatic];
p1 = Plot[-1 + 2 x Degree / π, {x, 0, 180}, PlotStyle -> {Gray, Dashed}];
p2 = Plot[3 - 2 x Degree / π, {x, 180, 360}, PlotStyle -> {Gray, Dashed}];
p3 = Plot[-5 + 2 x Degree / π, {x, 360, 540}, PlotStyle -> {Gray, Dashed}];
p4 = Plot[7 - 2 x Degree / π, {x, 540, 720}, PlotStyle -> {Gray, Dashed}];

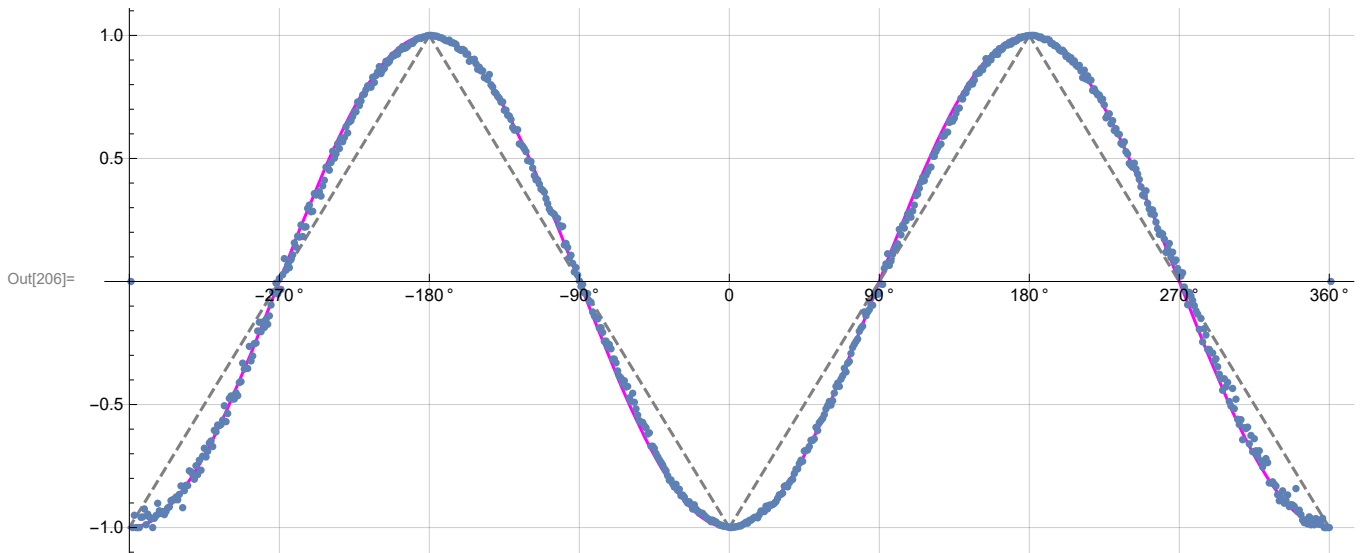
```

Comparing Mean Values with -Cosine Function and Computing Averages

```

In[206]:= Show[negcos, p1, p2, p3, p4, simulation]

```



In[207]:=

```

AveA = N[Sum[A[[i]], {i, m}]/m];
AveB = N[Sum[B[[i]], {i, m}]/m];
Print["AveA = ", AveA];
Print["AveB = ", AveB];
PAP = N[Sum[nAP[[i]], {i, trialDeg}]];
PBP = N[Sum[nBP[[i]], {i, trialDeg}]];
PAN = N[Sum[nAN[[i]], {i, trialDeg}]];
PBN = N[Sum[nBN[[i]], {i, trialDeg}]];
PA1 = PAP / (PAP + PAN);
PB1 = PBP / (PBP + PBN);
Print["P(A+) = ", PA1]
Print["P(B+) = ", PB1]
totAB = Sum[nPP[[i]] + nNN[[i]] + nPN[[i]] + nNP[[i]], {i, trialDeg}];
Print["Total Events = ", totAB]
PP = N[Sum[nPP[[i]], {i, trialDeg}]/totAB];
NN = N[Sum[nNN[[i]], {i, trialDeg}]/totAB];
PN = N[Sum[nPN[[i]], {i, trialDeg}]/totAB];
NP = N[Sum[nNP[[i]], {i, trialDeg}]/totAB];
Print["Ave ++ = ", PP]
Print["Ave -- = ", NN]
Print["Ave +- = ", PN]
Print["Ave -+ = ", NP]
CHSH = Abs[N[mean[[315]]] - N[mean[[225]]] + N[mean[[405]]] + N[mean[[45]]]];
Print["Approx. CHSH = ", CHSH]

AveA = -0.000115901

AveB = 0.0014283

P(A+) = 0.500033

P(B+) = 0.500685

Total Events = 1990970

Ave ++ = 0.250471

Ave -- = 0.249677

Ave +- = 0.249462

Ave -+ = 0.25039

Approx. CHSH = 2.74057

```

Product Calculation

```

In[231]:= (*q=0;
q2a=outA[All,4];
q2b=outB[All,4];
plotq=Table[{0,0},m];
angle=ConstantArray[0,m];
Do[If[λ2[[i]]==1,q=q2a[[i]]**q2b[[i]],q=q2b[[i]]**q2a[[i]]];
angle=theta[[i]]-361;
plotq[[i]]={angle,Re[q]},{i,m}]
sim=ListPlot[plotq,PlotMarkers→{Automatic,Small},AspectRatio→7/16,
Ticks→{{{ -360,-360°},{ -270,-270°},{ -180,-180°},{ -90,-90°},{ 0,0°},{ 90,90°},
{180,180°},{ 270,270°},{ 360,360°}},Automatic},GridLines→Automatic];
negcos1=Plot[-Cos[x Degree],{x,-360,360},PlotStyle→{Magenta}];
Show[sim,negcos1]*)

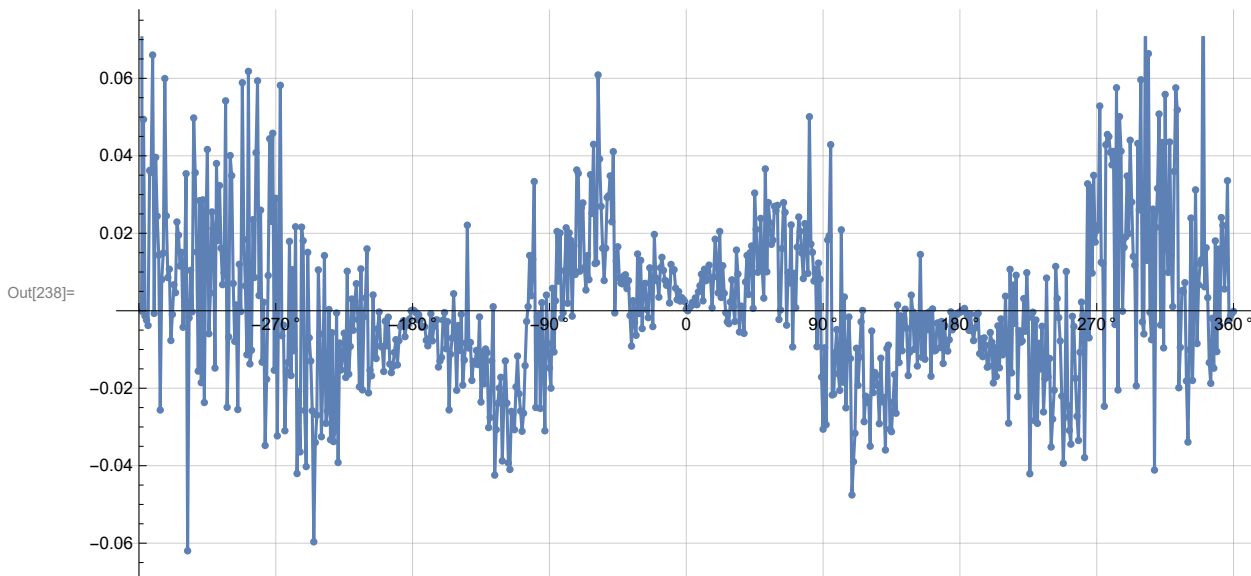
```

Blue is the correlation data and magenta is the -cosine curve for an exact match.

```

In[232]:= dev1 = ConstantArray[2, 720];
dev2 = ConstantArray[2, 720];
dev3 = ConstantArray[2, 720];
Do[dev1 = mean[[i]];
dev2[[i]] = {dev1, i}, {i, 720}]
devang = dev2[All, 2] - 361;
Do[dev3[[i]] = mean[[i]] + Cos[devang[[i]] Degree], {i, 720}]
ListPlot[N[dev3], PlotMarkers → {Automatic, Tiny}, Joined → True, AspectRatio → 1/2,
Ticks → {{{0, -360°}, {90, -270°}, {180, -180°}, {270, -90°}, {360, 0°}, {450, 90°},
{540, 180°}, {630, 270°}, {720, 360°}}, Automatic}, GridLines → Automatic]

```



```

In[239]:= N[Mean[Abs[dev3]]]
N[Mean[dev3]]

```

Out[239]= 0.0169498

Out[240]= 0.00273285