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Quantum Mechanical Prediction of the Singlet State with a Hidden Variable

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The following is a way of deriving the quantum mechanical (QM) prediction for the EPR-Bohm experiment [1] which has a hidden variable [2]. We have given in the Appendix a standard QM derivation for comparison. Consider a pair of spin one-half particles, moving freely after production in opposite directions, with particles 1 and 2 subject, respectively, to spin measurements along independently chosen unit directions $\mathbf{a}$ and $\mathbf{b}$, which can be located at a spacelike distance from each other. If initially the pair has vanishing total spin, then the pair's quantum mechanical spin state would be one of the following entangled chiral singlet states:

$$|\Psi_{\mathbf{n}\mathrm{R}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1 \otimes |\mathbf{n},\,-\rangle_2 \,-\, |\mathbf{n},\,-\rangle_1 \otimes |\mathbf{n},\,+\rangle_2\Big\}, \tag{1a}$$

$$\text{or} \quad |\Psi_{\mathbf{n}\mathrm{L}}\rangle = \frac{-1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1 \otimes |\mathbf{n},\,-\rangle_2 \,-\, |\mathbf{n},\,-\rangle_1 \otimes |\mathbf{n},\,+\rangle_2\Big\}, \tag{1b}$$

where $|\Psi_{\mathbf{n}\mathrm{R}}\rangle$ is the right-handed singlet state and $|\Psi_{\mathbf{n}\mathrm{L}}\rangle$ is the left-handed singlet state but viewed from the right handed perspective which elucidates the hidden variable $\lambda = \pm 1$ because we will have a $+1$ for the right handed state and -1 for the left handed state. And,

$$\boldsymbol{\sigma}\cdot\mathbf{n}\,|\mathbf{n},\,\pm\rangle \;=\; \pm\,|\mathbf{n},\,\pm\rangle \tag{2}$$

describing the quantum mechanical eigenstates in which the particles have spin "up" or "down" in units of $\hbar = 2$, with $\boldsymbol{\sigma}$ being the familiar Pauli spin "vector" $(\sigma_x,\,\sigma_y,\,\sigma_z)$.

Quantum mechanically the rotational invariance of the singlet state $|\Psi_{\mathbf{n}}\rangle$ ensures that the expectation values of the individual spin observables $\boldsymbol{\sigma}_1\cdot\mathbf{a}$ and $\boldsymbol{\sigma}_2\cdot\mathbf{b}$ are

$$\begin{aligned} \mathcal{E}_{q.m.}(\mathbf{a}) \;&=\; \langle\Psi_{\mathrm{R}\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\otimes\mathbb{1}\,|\Psi_{\mathrm{R}\mathbf{n}}\rangle \;=\; \langle\Psi_{\mathrm{R}\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\,|\Psi_{\mathrm{R}\mathbf{n}}\rangle \;=\; 0 \\ \text{and} \quad \mathcal{E}_{q.m.}(\mathbf{b}) \;&=\; \langle\Psi_{\mathrm{R}\mathbf{n}}|\,\mathbb{1}\otimes\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathrm{R}\mathbf{n}}\rangle \;=\; \langle\Psi_{\mathrm{R}\mathbf{n}}|\,\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathrm{R}\mathbf{n}}\rangle \;=\; 0\,, \end{aligned} \tag{3}$$

where $\mathbb{1}$ is the identity matrix and the results are the same for the left handed state.

We will also use the well known identities,

$$(i\boldsymbol{\sigma}\cdot\mathbf{a})\,(i\boldsymbol{\sigma}\cdot\mathbf{b}) \;=\; -\mathbf{a}\cdot\mathbf{b}\,\mathbb{1} \,-\, i\,\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}), \tag{4a}$$

$$(i\boldsymbol{\sigma}\cdot\mathbf{b})\,(i\boldsymbol{\sigma}\cdot\mathbf{a}) \;=\; -\mathbf{a}\cdot\mathbf{b}\,\mathbb{1} \,+\, i\,\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}), \tag{4b}$$

which follows from the non-commutativity of products of the Pauli matrices σ_j $(j = x, y, z)$ defined by the algebra

$$\sigma_j\sigma_k \;=\; \delta_{jk}\,\mathbb{1} + i\,\epsilon_{jkl}\,\sigma_l\,, \tag{5}$$

where δ_{jk} is the Kronecker delta, $i \equiv \sqrt{-1}$ is the unit imaginary, and ϵ_{jkl} is the Levi-Civita alternating symbol.

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Before we do the calculation for the correlation, we will do a conversion for the left handed state so that we have the correct ordering of the components in the calculation,

$$
\begin{aligned}
|\Psi_{\mathbf{n}\mathrm{L}}\rangle &= \frac{-1}{2}\Big\{\,|\mathbf{n},\,+\rangle_1 \otimes |\mathbf{n},\,-\rangle_2 \,-\, |\mathbf{n},\,-\rangle_1 \otimes |\mathbf{n},\,+\rangle_2\Big\}, \\
&= \frac{+1}{2}\Big\{\,|\mathbf{n},\,-\rangle_1 \otimes |\mathbf{n},\,+\rangle_2 \,-\, |\mathbf{n},\,+\rangle_1 \otimes |\mathbf{n},\,-\rangle_2\Big\},
\end{aligned} \tag{6}
$$

which changes the order of the up and down states effectively reversing the order of the components in the calculation. Which is then converted like this for the bra-ket calculation,

$$
\langle\Psi_{\mathbf{n}\mathrm{L}}|(i\boldsymbol{\sigma}\cdot\mathbf{a})(i\boldsymbol{\sigma}\cdot\mathbf{b})|\Psi_{\mathbf{n}\mathrm{L}}\rangle = \langle\Psi_{\mathbf{n}\mathrm{R}}|(i\boldsymbol{\sigma}\cdot\mathbf{b})(i\boldsymbol{\sigma}\cdot\mathbf{a})|\Psi_{\mathbf{n}\mathrm{R}}\rangle, \tag{7}
$$

so that we are only performing the calculation in the right handed basis.

We are now ready to do the calculation for the correlation,

$$
\mathcal{E}_{L.R.}(\mathbf{a},\,\mathbf{b}) = \lim_{n>>1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\{\langle\Psi_{\mathbf{n}\mathrm{R}}|(i\boldsymbol{\sigma}\cdot\mathbf{a})(i\boldsymbol{\sigma}\cdot\mathbf{b})|\Psi_{\mathbf{n}\mathrm{R}}\rangle \;\mathrm{if}\,(\lambda^k = +1)\,\mathrm{else}\,\langle\Psi_{\mathbf{n}\mathrm{R}}|(i\boldsymbol{\sigma}\cdot\mathbf{b})(i\boldsymbol{\sigma}\cdot\mathbf{a})|\Psi_{\mathbf{n}\mathrm{R}}\rangle\}\right] \tag{8}
$$

$$
= \frac{1}{2}\,\langle\Psi_{\mathbf{n}\mathrm{R}}|(i\boldsymbol{\sigma}\cdot\mathbf{a})(i\boldsymbol{\sigma}\cdot\mathbf{b})|\Psi_{\mathbf{n}\mathrm{R}}\rangle \,+\, \frac{1}{2}\,\langle\Psi_{\mathbf{n}\mathrm{R}}|(i\boldsymbol{\sigma}\cdot\mathbf{b})(i\boldsymbol{\sigma}\cdot\mathbf{a})|\Psi_{\mathbf{n}\mathrm{R}}\rangle \tag{9}
$$

$$
= \frac{1}{2}\,\langle\Psi_{\mathbf{n}\mathrm{R}}|(-\mathbf{a}\cdot\mathbf{b}\,\mathbb{1} -\, i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}))|\Psi_{\mathbf{n}\mathrm{R}}\rangle \,+\, \frac{1}{2}\,\langle\Psi_{\mathbf{n}\mathrm{R}}|(-\mathbf{a}\cdot\mathbf{b}\,\mathbb{1} +\, i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}))|\Psi_{\mathbf{n}\mathrm{R}}\rangle \tag{10}
$$

$$
= -\frac{1}{2}\,\mathbf{a}\cdot\mathbf{b} \,-\, \frac{1}{2}\,\mathbf{a}\cdot\mathbf{b} \tag{11}
$$

$$
= -\,\mathbf{a}\cdot\mathbf{b}. \tag{12}
$$

where we have used the Pauli identities, and the cross product terms reduce to zero because of the rotational invariance of the singlet state. It is easy to see from the cross product terms that we indeed have left and right handed components as they are pointing in opposite directions.

When $\lambda = +1$ we will have the right handed consequence and when $\lambda = -1$ we will have the left handed consequence. So what is the proof that the singlet has such orientations as left and right? The fact is that in three dimensional space anything bigger than a mathematical point can have parity due to the very nature of that space [3]. Another notion is that the singlet has a parallelized 3-sphere topology [4]. This is very easy to demonstrate via the Pauli algebra. So we started by assuming chirality but now it is hopefully easy to see that the chirality of the singlet state is a fact. Plus it is expected and accepted that certain objects in Nature have a handedness. So, quantum mechanics could easily have a hidden variable due to chirality and is finally complete after many have tried.

Appendix: Standard QM Derivation

Considered a pair of spin one-half particles, moving freely after production in opposite directions, with particles 1 and 2 subject, respectively, to spin measurements along independently chosen unit directions $\mathbf{a}$ and $\mathbf{b}$, which can be located at a spacelike distance from each other. If initially the pair has vanishing total spin, then its quantum mechanical (QM) spin state would be the entangled singlet state

$$
|\Psi_{\mathbf{n}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1 \otimes |\mathbf{n},\,-\rangle_2 \,-\, |\mathbf{n},\,-\rangle_1 \otimes |\mathbf{n},\,+\rangle_2\Big\}\,, \tag{13}
$$

with $\mathbf{n}$ indicating an arbitrary unit direction, and

$$
\boldsymbol{\sigma}\cdot\mathbf{n}\,|\mathbf{n},\,\pm\rangle \,=\, \pm\,|\mathbf{n},\,\pm\rangle \tag{14}
$$

describing the quantum mechanical eigenstates in which the particles have spin "up" or "down" in units of $\hbar = 2$. Here $\boldsymbol{\sigma}$ is the familiar Pauli spin "vector" $(\sigma_x,\,\sigma_y,\,\sigma_z)$. Our interest lies in comparing the quantum predictions of spin correlations between the two remote subsystems with those derived from any locally causal theory.

Now, quantum mechanically the rotational invariance of the state $|\Psi_{\mathbf{n}}\rangle$ ensures that the expectation values of the individual spin observables $\boldsymbol{\sigma}_1 \cdot \mathbf{a}$ and $\boldsymbol{\sigma}_2 \cdot \mathbf{b}$ are

$$\begin{aligned} \mathcal{E}_{q.m.}(\mathbf{a}) &= \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1 \cdot \mathbf{a} \otimes \mathbb{1}\,|\Psi_{\mathbf{n}}\rangle = \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1 \cdot \mathbf{a}\,|\Psi_{\mathbf{n}}\rangle = 0 \\ \text{and}\quad \mathcal{E}_{q.m.}(\mathbf{b}) &= \langle\Psi_{\mathbf{n}}|\,\mathbb{1} \otimes \boldsymbol{\sigma}_2 \cdot \mathbf{b}\,|\Psi_{\mathbf{n}}\rangle = \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_2 \cdot \mathbf{b}\,|\Psi_{\mathbf{n}}\rangle = 0\,, \end{aligned} \tag{15}$$

where $\mathbb{1}$ is the identity matrix. The expectation value of the joint observable $\boldsymbol{\sigma}_1 \cdot \mathbf{a} \otimes \boldsymbol{\sigma}_2 \cdot \mathbf{b}$, on the other hand, is

$$\mathcal{E}_{q.m.}(\mathbf{a},\,\mathbf{b}) = \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1 \cdot \mathbf{a} \,\otimes\, \boldsymbol{\sigma}_2 \cdot \mathbf{b}\,|\Psi_{\mathbf{n}}\rangle = -\,\mathbf{a} \cdot \mathbf{b}\,, \tag{16}$$

regardless of the relative distance between the two remote locations represented by the unit vectors $\mathbf{a}$ and $\mathbf{b}$. The last result can be derived using the well known identity with either right or left chirality,

$$(+i\,\boldsymbol{\sigma} \cdot \mathbf{a})\,(+i\,\boldsymbol{\sigma} \cdot \mathbf{b}) = -\,\mathbf{a} \cdot \mathbf{b}\,\mathbb{1} - i\,\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b}) \tag{17a}$$

$$\text{or}\quad (-i\,\boldsymbol{\sigma} \cdot \mathbf{a})\,(-i\,\boldsymbol{\sigma} \cdot \mathbf{b}) = -\,\mathbf{a} \cdot \mathbf{b}\,\mathbb{1} + i\,\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b})\,, \tag{17b}$$

which follows from the non-commutativity of products of the Pauli matrices σ_j $(j = x, y, z)$ defined by the algebra

$$\sigma_j \sigma_k = \delta_{jk}\,\mathbb{1} \pm i\,\epsilon_{jkl}\,\sigma_l\,, \tag{18}$$

where δ_{jk} is the Kronecker delta, $i \equiv \sqrt{-1}$ is the unit imaginary, and ϵ_{jkl} is the Levi-Civita alternating symbol.

To prove the result (16), simply substitute the right-hand-side of the identity (17a) or (17b) into Eq. (16) and obtain

$$\begin{aligned} \langle\Psi_{\mathbf{n}}|-\,\mathbf{a} \cdot \mathbf{b}\,\mathbb{1} \mp i\,\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b})\,|\Psi_{\mathbf{n}}\rangle &= -\,\mathbf{a} \cdot \mathbf{b}\,\langle\Psi_{\mathbf{n}}|\,\mathbb{1}\,|\Psi_{\mathbf{n}}\rangle \mp i\,\langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b})\,|\Psi_{\mathbf{n}}\rangle \\ &= -\,\mathbf{a} \cdot \mathbf{b} \mp 0\,, \end{aligned} \tag{19}$$

where the last equality follows from using eq. (15), which in turn follows from the rotational invariance of the state (13). One might wonder why this derivation gives the same result as the previous derivation with all right handed singlets. It is because this derivation simply ignores that half of the singlets are left handed and counts them as right handed.

A statement of intent by Jay R. Yablon:

In preparation for the symposium regarding Joy Christian's efforts to devise a local realistic hidden variable model of strong singlet correlations as a counterexample to Bells Theorem, the moderator / mediator Jay R. Yablon articulated the viewpoint that geometric algebra (GA) which Joy uses in his model, and Pauli matrices with eigenstates and eigenvalues customarily used for describing these correlations, provide two different mathematical languages which can be used to describe the strong singlet correlations, with identifiable transformations (translations) between them. Importantly, Jay starts from the view that if we hypothesize that neither language is fundamentally flawed, then the underlying natural reality must be the same no matter which of these two languages we choose. In short, assuming unflawed languages, nature is and must be invariant with respect to the language we use to describe her. Based on this, Jay began, and is presently working on a more extensive paper which seeks to closely study the standard quantum mechanical description of strong correlations using Pauli matrices and their states and values (and in some cases, values which cannot be simultaneously observed owing to the uncertainty principle), both to stand on its own right, and to provide another language for analyzing the local realistic model which Joy Christian has used to describe these correlations. In the process, it will also become possible to test the validity of the hypothesis that neither language is flawed, and / or to correct any flawed interpretations of these languages. Some of the specifics laid out in the draft paper being posted here may be modified as this deeper study advances.

[1] D. Bohm and Y. Aharonov, *Discussions of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky*, Phys. Rev. **108**, 1079 (1957).

[2] J. Christian, *Quantum correlations are weaved by the spinors of the Euclidean primitives*. R. Soc. Open Sci. 5, 180526; doi:10.1098/rsos.180526 (2018), arXiv:1806.02392; See also: Eight-dimensional Octonion-like but Associative Normed Division Algebra. https://hal.archives-ouvertes.fr/hal-01933757v1 (2018).

[3] For a good explanation see J. Yablon, *Directly-observable versus observably-consequential elements of reality and the uncertainty principle: Why quantum mechanics properly understood is a realistic and complete hidden variable theory of the natural world*, https://jayryablon.files.wordpress.com/2019/06/singlet-state-4.1-1.pdf, Section 2 (2019).

[4] J. Christian, *Local causality in a Friedmann-Robertson-Walker spacetime*, arXiv: 1405.2355 (2014).